1.8. Properties of Kuhn chain :

For N >> 1, the Kuhn chain is also called the GAUSSIAN chain.

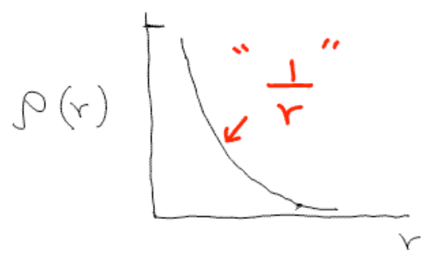
(a) Mean square end-to-end distance: $\langle R^2 \rangle_0 = N\ell^2$

(b) Mean square radius of gyration: $R_g^2 = \dfrac{N\ell^2}{6}$

(c) Probability of the end-to-end distance at $\vec{R}$:

$$P(\vec{R}) = \left(\frac{3}{2\pi N\ell^2}\right)^{3/2} e^{-\frac{3R^2}{2N\ell^2}}$$

(d) Segmental density profile:



Long-ranged correlation due to chain connectivity $\Longrightarrow$ "$\frac{1}{r}$"

(e) Free energy of a chain with the end-to-end distance $\vec{R}$:

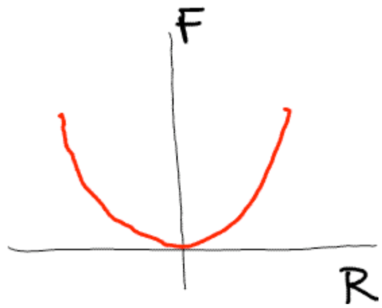
$$F(\vec{R}) = E - TS$$

$$S = k_B \ln P$$

$$= -\frac{3k_B R^2}{2N\ell^2} + \frac{3k_B}{2}\ln\left(\frac{3}{2\pi N\ell^2}\right)$$

E = N ε (ε is the energy of a segment)

$$F(\vec{R}) = \text{constant} + \frac{3k_BT}{2N\ell^2}R^2$$



(f) Force to pull a chain end:

$$\vec{f} = \frac{\delta F(\vec{R})}{\delta \vec{R}}$$

$$\vec{f} = \frac{3k_BT}{N\ell^2}\vec{R}$$

Each chain appears to be a Hookean spring. Since the spring constant is proportional to T/N, the chain is highly elastic, and consequently the societal benefits of polymers.

1.9. Home work problem:

1.9.1. Consider a Gaussian chain with 1000 Kuhn segments, each with the Kuhn segment length of 1 nm. Calculate the spring constant, in units of pN/nm, for stretching the chain at T = 300^{o}K.